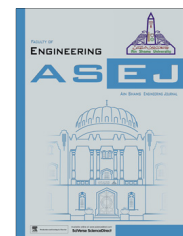




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Effect of partial slip on hydromagnetic flow over a porous stretching sheet with non-uniform heat source/sink, thermal radiation and wall mass transfer



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Abstract In this work, we have investigated the effect of partial slip on hydromagnetic boundary layer flow in porous medium over a stretching surface with space and temperature dependent internal heat generation/absorption, thermal radiation and wall mass transfer (suction/blowing). The basic boundary layer equations for momentum and heat transfer, which are non-linear partial differential equations are converted into non-linear ordinary differential equations by means of similarity transformations. The dimensionless governing equations for this investigation are solved analytically using hypergeometric functions. The results are carried out for prescribed surface temperature (PST) and prescribed power law surface heat flux (PHF). The effect of partial slip parameter with magnetic parameter, porosity parameter, wall mass transfer parameter (suction/blowing), space and temperature dependent internal heat generation/absorption parameter, Prandtl number and radiation parameter on velocity and temperature distributions are depicted graphically and are analyzed in detail.

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1. Introduction

Due to the numerous applications in industrial manufacturing processes, the problem of flow and heat transfer due to stretching surfaces has attracted the attention of researchers for the past four decades being a subject of considerable interest in

the contemporary literature [1–5]. Some of the application areas are hot rolling, paper production, metal spinning, drawing plastic films, glass blowing, continuous casting of metals and spinning of fibers, etc. [6]. In all these cases, the quality of the final product depends on the rate of heat transfer at the stretching surface. Many of the flow properties were investigated by the followers [7–11] using no-slip condition on the wall. However, as stated in [12], when the fluid is particulate such as emulsions, suspensions, foams and polymer solutions, the no-slip condition is inadequate. In such cases the suitable boundary condition is the partial slip. Wang [13] discussed the partial slip effects on the planar stretching flow. Partial slip flow over a stretching sheet is investigated by many researchers [14–18].

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and v are the fluid tangential velocity and normal velocity, respectively (see Fig. 1).

The equation governing the problem under consideration is given by

$$u_x + v_y = 0 \quad (1)$$

$$uu_x + vv_y = \vartheta u_{yy} - \frac{\vartheta}{K}u - \frac{\sigma B_0^2 u}{\rho} \quad (2)$$

where ϑ is the kinematic viscosity, K is the permeability of the porous medium, ρ is the density, σ is the electrical conductivity and B_0 is the uniform magnetic field in the y -direction.

The boundary conditions for the velocity fields are of the form

$$u(x, 0) = ax + lu_y(0), \quad v(y, 0) = v_w, \quad u(x, \infty) = 0 \quad (3)$$

To facilitate the analysis, we introduce the subsequent conventional similarity transformations and dimensionless variables η and $f(\eta)$

$$u = axf_\eta(\eta), \quad v = -(a\vartheta)^{\frac{1}{2}}f(\eta), \quad \eta = \left(\frac{a}{\vartheta}\right)^{\frac{1}{2}}y \quad (4)$$

Using (4), Eq. (1) is trivially satisfied and Eqs. (2) and (3) take the form:

$$ff_{\eta\eta} + ff_{\eta\eta} - f_\eta^2 - \lambda f_\eta - Mn f_\eta = 0 \quad (5)$$

with corresponding boundary conditions

$$\begin{aligned} f(\eta) &= s, \quad f_\eta(\eta) = 1 + Lf_{\eta\eta}(0) \quad \text{at} \quad \eta = 0 \\ f_\eta(\eta) &\rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty \end{aligned} \quad (6)$$

Here the subscript η denotes differentiation with respect to η , s is the wall mass transfer [$s < 0$ (blowing), $s > 0$ (suction)], $\lambda = \frac{\sigma B_0^2}{aK}$ is the porosity parameter, $Mn = \frac{\sigma B_0^2}{\rho a}$ is the magnetic parameter and $L = l\sqrt{\frac{a}{\vartheta}}$ is the partial slip parameter.

The solution of Eq. (5) with boundary condition (6) is obtained as,

$$f(\eta) = s + X \left(\frac{1 - e^{-\alpha\eta}}{\alpha} \right) \quad (7)$$

where

$$X = \frac{1}{L\alpha + 1}$$

$$\alpha = -\left(\frac{1 - Ls}{3L}\right) - \frac{(2)^{\frac{1}{3}}\alpha_1}{3L(\alpha_2 + \sqrt{4\alpha_1^3 + \alpha_2^2})^{\frac{1}{3}}} + \frac{(\alpha_2 + \sqrt{4\alpha_1^3 + \alpha_2^2})^{\frac{1}{3}}}{(2)^{\frac{1}{3}}3L}$$

and

$$\alpha_1 = L^2(-3Mn - 3\lambda - s^2) - Ls - 1$$

$$\alpha_2 = L^3(9s\lambda + 9sMn + 2s^3) + L^2(27 + 18Mn + 18\lambda + 3s^2) - 3Ls - 2$$

Using the solution (7) in Eq. (4), the velocity components obtained in the form

$$u = ax(Xe^{-\alpha\eta}) \quad \text{and} \quad v = -\sqrt{a\vartheta} \left(s + X \left(\frac{1 - e^{-\alpha\eta}}{\alpha} \right) \right) \quad (8)$$

The wall shearing stress on the surface of the stretching sheet is given by

$$\tau_w = \left[v \left(\frac{\partial u}{\partial y} \right) \right]_{y=0} \quad (9)$$

The local skin-friction coefficient or the frictional drag is given by

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho u_w^2} = 2Re_x^{-1/2}f_{\eta\eta}(0) \quad (10)$$

where $Re_x = \frac{xu_w}{\vartheta}$ is the Reynolds number.

3. Heat transfer analysis

The governing thermal boundary layer equation in the presence of non-uniform internal heat source/sink and thermal radiation for two dimensional flow problem is given by

$$\rho c_p (uT_x + vT_y) = kT_{yy} - q_r + q''' \quad (11)$$

where k is the thermal conductivity, ρ is the density, T is temperature, c_p is the specific heat of constant pressure and q''' is the space and temperature dependent internal heat generation/absorption (non-uniform heat source/sink) [24] which can be expressed in simplest form as

$$q''' = \left(\frac{ku_w(x)}{x\vartheta} \right) [A^*(T_w - T_\infty)f_\eta + B^*(T - T_\infty)] \quad (12)$$

where A^* and B^* are parameters of the space and temperature dependent internal heat generation/absorption. It is to be noted that $A^* > 0$ and $B^* > 0$ correspond to internal heat generation while $A^* < 0$ and $B^* < 0$ correspond to internal absorption.

The Rosseland approximation for radiation [25] heat flux has given by

$$q_r = -\frac{4\sigma^*}{3k^*}T_y^4 \quad (13)$$

where σ^* is the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. Further, we assume that the temperature difference within in flow is such that T^4 may be expanded in a Taylor series. Hence expanding T^4 about T_∞ and neglecting higher order terms we get

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (14)$$

Using Eqs. (13) and (14) in (11), we obtain

$$\rho c_p (uT_x + vT_y) = kT_{yy} - \frac{16\sigma^* T_\infty^3}{3k^*} T_{yy} + q''' \quad (15)$$

The solution of Eq. (15) is found using the two types of general heating processes such as prescribed surface temperature (PST) and prescribed power law of surface heat flux (PHF), condition as described below.

3.1. The prescribed surface temperature (PST CASE)

The boundary condition in the PST case is given by

$$T = T_w = T_\infty + A \left(\frac{x}{l} \right)^2 \quad \text{at} \quad y = 0 \quad (16)$$

$$T \rightarrow T_\infty \quad \text{as} \quad y \rightarrow \infty$$

where T_w is the temperature of the sheet, T_∞ is the temperature of the fluid far away from the sheet and l is the characteristic length.

Define the non-dimensional temperature $\theta(\eta)$ as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (17)$$

Now, we make use of the transformations given by Eqs. (4), (12) and (17) in Eq. (15). This leads to the non-dimensional form of temperature equation as follows:

$$\omega\theta_{\eta\eta} + Prf\theta_\eta - 2Prf_\eta\theta + A^*f_\eta + B^*\theta = 0 \quad (18)$$

where $Pr = \frac{\mu c_p}{k}$ is the Prandtl number, $N = \frac{k^*k}{4\sigma^*T_\infty^3}$ is radiation parameter and $\omega = (3N + 4)(3N)^{-1}$.

Consequently the boundary condition Eq. (16) takes the form,

$$\begin{aligned} \theta(\eta) &= 1 \quad \text{at} \quad \eta = 0 \\ \theta(\eta) &\rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty \end{aligned} \quad (19)$$

The solution of Eq. (18), subject to boundary condition of Eq. (19) can be obtained in terms of hypergeometric function as

$$\begin{aligned} \theta(\eta) &= c_1 e^{-\alpha\left(\frac{p+q}{2}\right)\eta} M\left(\frac{p+q-4}{2}, 1+q, \frac{-Pr}{\alpha^2\omega} X e^{-\alpha\eta}\right) \\ &\quad - c_3 e^{-2\alpha\eta} - c_2 e^{-\alpha\eta} \end{aligned} \quad (20)$$

where

$$\begin{aligned} p &= Pr\left(\frac{s}{\omega\alpha} + \frac{X}{\omega\alpha^2}\right) \\ q &= \sqrt{p^2 - 4\frac{B^*}{\omega\alpha^2}} \\ c_1 &= (1 + c_3 + c_2) \left(M\left(\frac{p+q-4}{2}, 1+q, \frac{-Pr}{\alpha^2\omega} X\right) \right)^{-1} \\ c_2 &= \frac{XA^*}{\omega\alpha^2\left(1 - p + \frac{B^*}{\omega\alpha^2}\right)} \\ \text{and } c_3 &= \frac{X^2 A^* Pr}{\omega^2 \alpha^4 \left(1 - p + \frac{B^*}{\omega\alpha^2}\right) \left(4 - 2p + \frac{B^*}{\omega\alpha^2}\right)} \end{aligned}$$

The relation $\frac{4B^*}{\omega\alpha^2} \leq p^2$ must be satisfied in order to have real value of q . The non-dimensional wall temperature gradient obtained from (21) as:

$$\begin{aligned} \theta_\eta(0) &= -c_1 \alpha \left(\frac{p+q}{2}\right) M\left(\frac{p+q-4}{2}, 1+q, \frac{-Pr}{\alpha^2\omega} X\right) \\ &\quad + \frac{c_1 Pr X}{\alpha\omega} \left(\frac{p+q-4}{2(1+q)}\right) M\left(\frac{p+q-2}{2}, 2+q, \frac{-Pr}{\alpha^2\omega} X\right) \\ &\quad + 2\alpha c_3 + c_2 \alpha \end{aligned} \quad (21)$$

The local heat flux can be expressed as

$$q_w = -k(T_y)_{y=0} = -k\sqrt{\frac{a}{\vartheta}}(T_w - T_\infty)\theta_\eta(0) \quad (22)$$

3.2. Prescribed power law of surface heat flux (PHF CASE)

The boundary condition in case of PHF is given by

$$-kT_y = q_w = D\left(\frac{X}{l}\right)^2 \quad \text{at} \quad y = 0, \quad T \rightarrow T_\infty \quad \text{as} \quad y \rightarrow \infty \quad (23)$$

where D is constant, k and l are defined earlier.

We now define a dimensionless, scaled temperature $g(\eta)$ as

$$g(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (24)$$

and make use of the transformation given by Eq. (4). This leads to the following non-dimensional form of Eq. (15) for temperature

$$\omega g_{\eta\eta} + Prfg_\eta - 2Prf_\eta g + A^*f_\eta + B^*g = 0 \quad (25)$$

The corresponding boundary condition takes the form

$$\begin{aligned} g_\eta(\eta) &= -1 \quad \text{at} \quad \eta = 0 \\ g(\eta) &\rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty \end{aligned} \quad (26)$$

The analytic solution of Eq. (26), subject to the boundary condition Eq. (27), is obtained in the following form of hypergeometric function of the similarity variable

$$\begin{aligned} g(\eta) &= c_4 e^{-\alpha\left(\frac{p+q}{2}\right)\eta} M\left(\frac{p+q-4}{2}, 1+q, \frac{-Pr}{\alpha^2\omega} X e^{-\alpha\eta}\right) \\ &\quad - c_3 e^{-2\alpha\eta} - c_2 e^{-\alpha\eta} \end{aligned} \quad (27)$$

Here p, q, c_2 and c_3 are as defined earlier in the PST case and c_4 is given by

$$c_4 = \frac{-1 - 2\alpha c_3 - \alpha c_2}{\left[-\alpha\left(\frac{p+q}{2}\right) M\left(\frac{p+q-4}{2}, 1+q, \frac{-PrX}{\alpha^2\omega}\right) + \left(\frac{p+q-4}{2(1+q)}\right) \left(\frac{PrX}{\alpha\omega}\right) M\left(\frac{p+q-2}{2}, 2+q, \frac{-PrX}{\alpha^2\omega}\right)\right]}$$

The non-dimensional wall temperature derived from Eq. (28) read as:

$$g(0) = c_4 M\left(\frac{p+q-4}{2}, 1+q, \frac{-PrX}{\alpha^2\omega}\right) - c_3 - c_2 \quad (28)$$

Table 1 Comparison values of $-\theta_\eta(0)$ and $g(0)$ in PST and PHF cases.

Mn	Pr	$-\theta_\eta(0)$		$g(0)$	
		Turkyilmazoglu [21] with $L = \lambda = 0, s = A^* = B^* = 0$ without radiation	Present value with $L = \lambda = 0, s = A^* = B^* = 0$ without radiation	Turkyilmazoglu [21] with $L = \lambda = 0, s = A^* = B^* = 0$ without radiation	Present value with $L = \lambda = 0, s = A^* = B^* = 0$ without radiation
0		3.31648	3.31648	—	0.30152
	5	4.79687	4.79687	—	0.20846
	10				
1		1.21577	1.21577	0.82252	0.82252
	1	—	3.20721	—	0.31179
	5	—	4.69021	—	0.21321
	10				

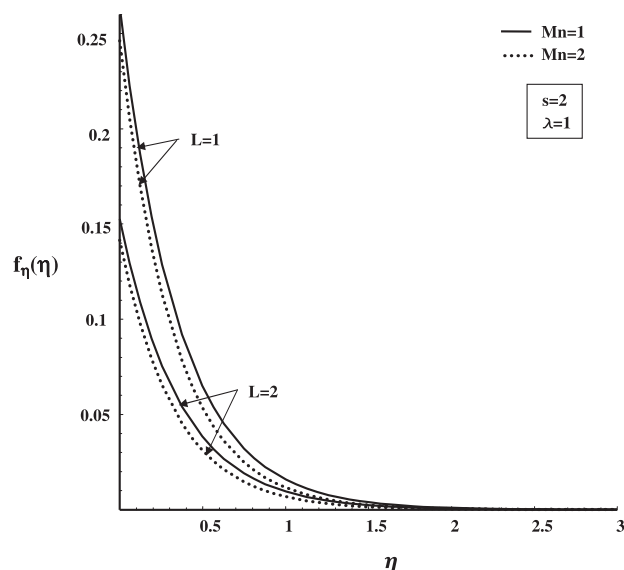


Figure 2 Effect of slip parameter L with magnetic parameter Mn on velocity profile.

The expression for wall temperature in dimensional form is:

$$T_w = T_\infty + \frac{q_w}{k} \sqrt{\frac{\eta}{a}} g(0) \quad (29)$$

4. Result and discussion

In the present work, we have analyzed the effect of partial slip on hydromagnetic boundary layer flow over a porous stretching sheet with space and temperature dependent internal heat generation/absorption, thermal radiation and suction/blowing effects. Analytical solutions are obtained in terms of hypergeometric function. Thermal transport is analyzed for prescribed surface temperature (PST) and prescribed power law surface

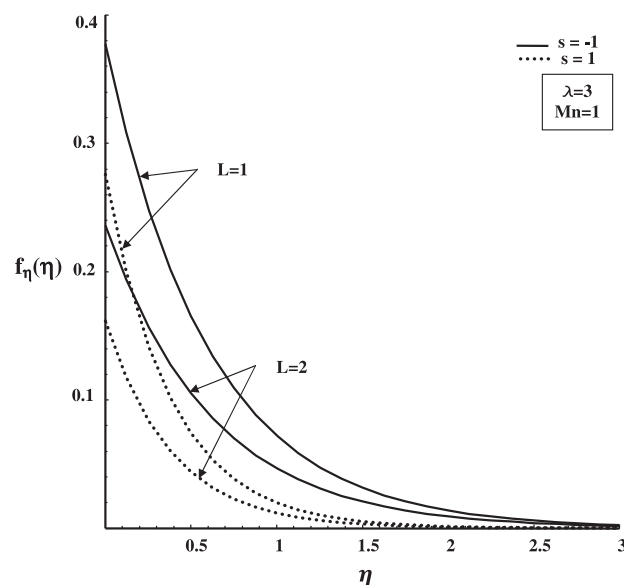


Figure 4 Effect of slip parameter L with wall mass transfer parameter s on velocity profile.

heat flux (PHF) cases. In order to validate our present work, we have compared our results with those of Turkiymazoglu [21] for $-\theta_\eta(0)$ and $g(0)$ for PST and PHF cases respectively, in the absence of partial slip parameter (L), Porosity parameter (λ), mass transfer parameter (s), non-uniform heat source/sink parameter (A^* , B^*) and radiation parameter (N). The comparison in the above cases is found to be in excellent agreement as shown in Table 1.

The effect of magnetic parameter (Mn) with partial slip parameter (L) on the velocity profile $f_\eta(\eta)$ is presented in Fig. 2. It is noted that the rising of magnetic parameter reduces the velocity profile. This is due to the fact the increase in Mn , Lorentz force increases and it produces more resistance to the

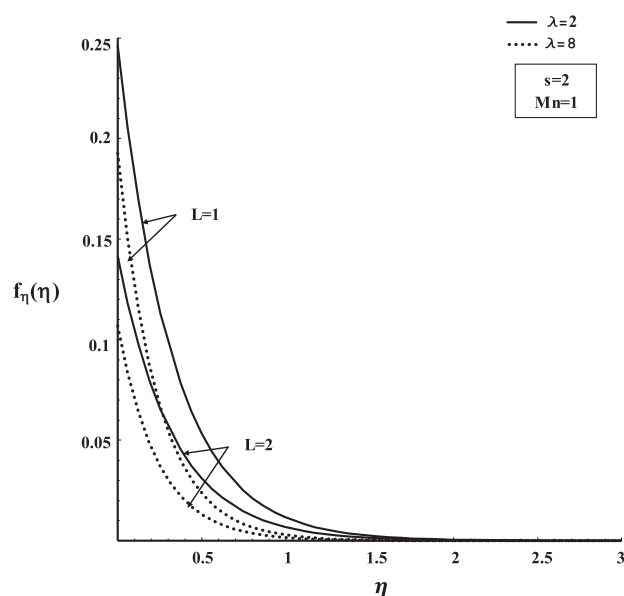


Figure 3 Effect of slip parameter L with porosity parameter λ on the velocity profile.

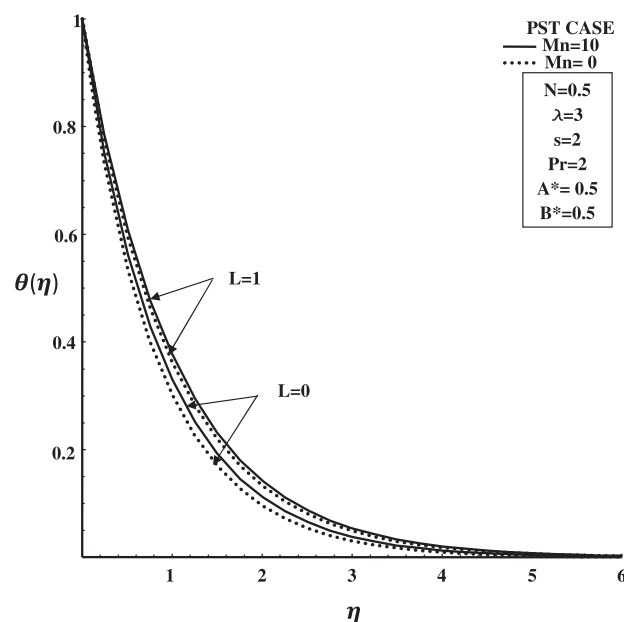


Figure 5a Effect of slip parameter L with magnetic parameter Mn on the temperature profile in PST case.

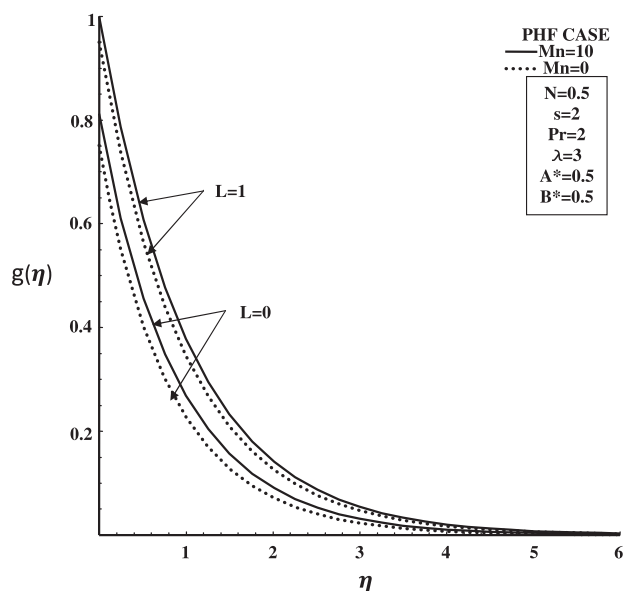


Figure 5b Effect of slip parameter L with magnetic parameter Mn on the temperature profile in PHF case.

flow. The presence of transverse magnetic field leads to decrease the momentum boundary layer thickness. The velocity profile of the fluid increases as partial slip parameter decreases. When slip occurs, the flow velocity near the sheet is no longer equal to the stretching velocity of the sheet. With the increase in L , such slip velocity increases and consequently fluid velocity decreases because under the slip condition, the pulling of the stretching sheet can be only partly transmitted to the fluid. The presence of transverse magnetic field and the partial slip lead to decrease the momentum boundary layer thickness.

Fig. 3 depicts the effect of partial slip parameter with porosity parameter on $f_\eta(\eta)$. It is clear from the figure the velocity

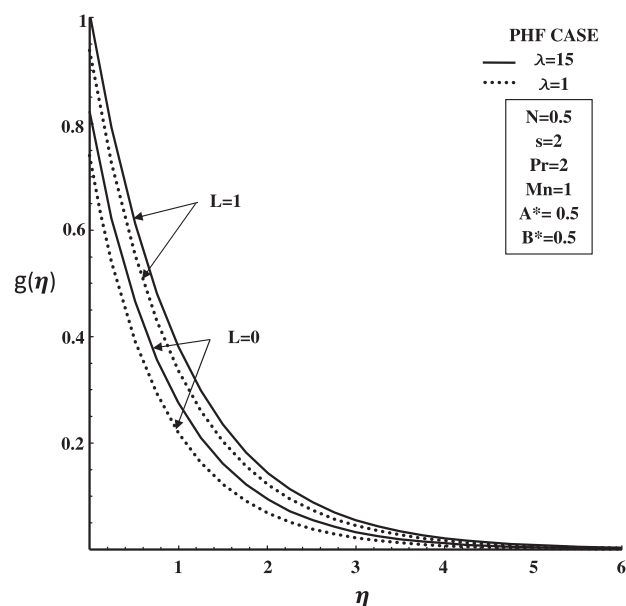


Figure 6b Effect of slip parameter L with porosity parameter λ on the temperature profile in PHF case.

profile decreases with the increasing of porosity parameter and the partial slip parameter. The presence of a porous medium causes higher restriction to the fluid flow which, in turn, slows its motion. Increasing of both the parameters lead to thinning of the momentum boundary layer.

The effect of wall mass transfer (s) parameter with partial slip parameter on velocity profile is shown in Fig. 4. It is observed that velocity decreases significantly with increasing suction parameter whereas fluid velocity is found to increase with blowing. It is observed that, when the wall suction ($s > 0$) is considered, this causes a decrease in the boundary layer thickness and the velocity field is reduced. Opposite

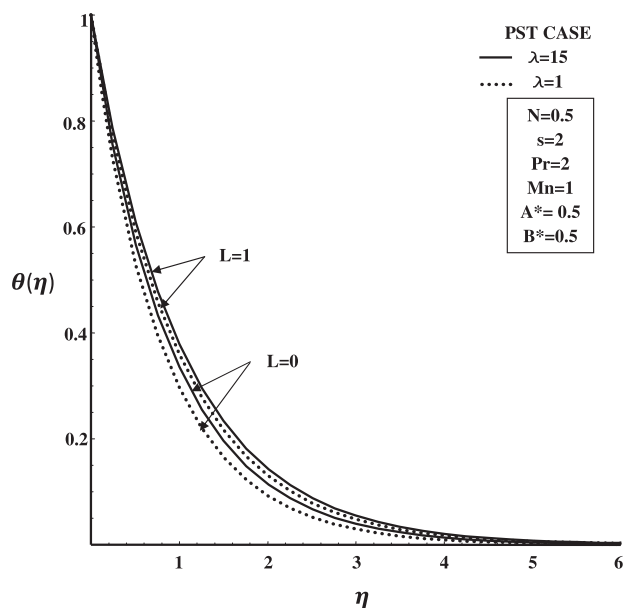


Figure 6a Effect of slip parameter L with porosity parameter λ on the temperature profile in PST case.

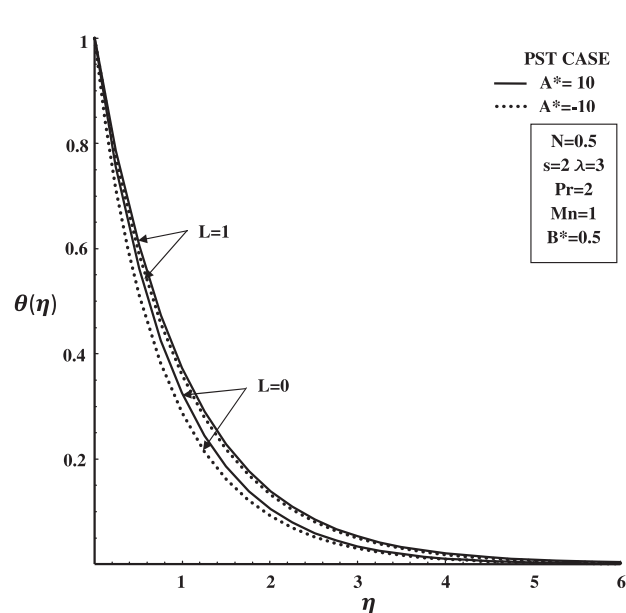


Figure 7a Effect of slip parameter L with space dependent heat source/sink coefficient A^* on the temperature profile in PST case.

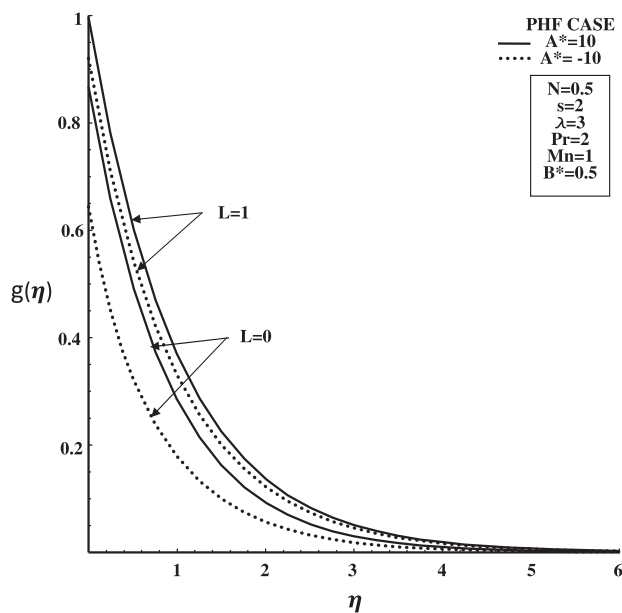


Figure 7b Effect of slip parameter L with space dependent heat source/sink coefficient A^* on the temperature profile in PHF case.

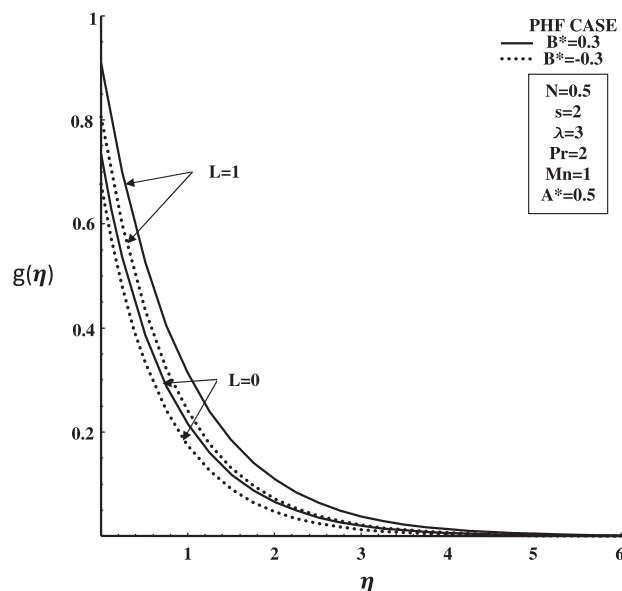


Figure 8b Effect of slip parameter L with temperature dependent heat source/sink coefficient B^* on the temperature profile in PHF case.

behavior is noted for blowing ($s < 0$). The velocity profile decreases as slip parameter increases.

Figs. 5a and 5b demonstrate the effect of magnetic parameter with the partial slip parameter on the temperature profile in PST and PHF cases respectively. The presence of magnetic parameter and partial slip parameter lead to increase the temperature profile in both PST (Fig. 5a) and PHF (Fig. 5b) cases. The combined effects of magnetic and slip parameters lead to increase the thickness of hydromagnetic boundary layer. The fluid becomes warmer in the presence of magnetic field.

The effect of porosity parameter with partial slip parameter on the temperature profiles for both PST and PHF cases is presented in Figs. 6a and 6b. The increasing of the partial slip parameter and the porosity parameter increases the dimensionless temperature profile in both PST (Fig. 6a) and PHF (Fig. 6b) cases. The effect of porosity parameter with partial slip parameter has the same effect as the magnetic parameter with partial slip parameter on the hydromagnetic boundary layer.

The effect of space and temperature dependent heat source/sink with partial slip parameter on PST and PHF cases are

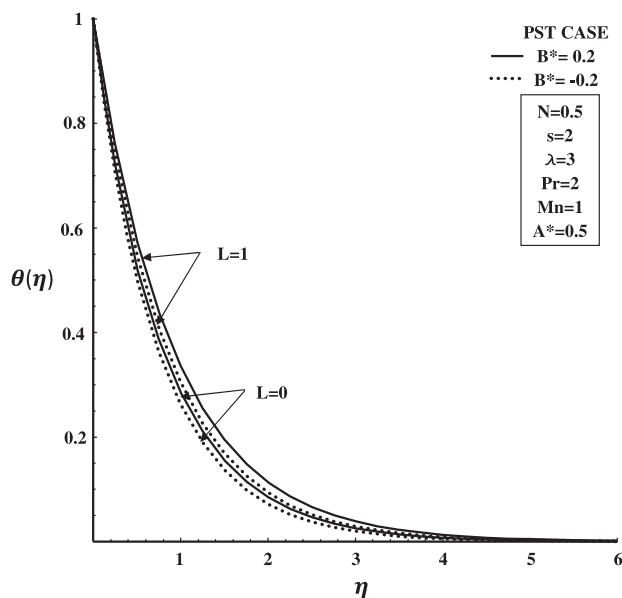


Figure 8a Effect of slip parameter L with temperature dependent heat source/sink coefficient B^* on the temperature profile in PST case.

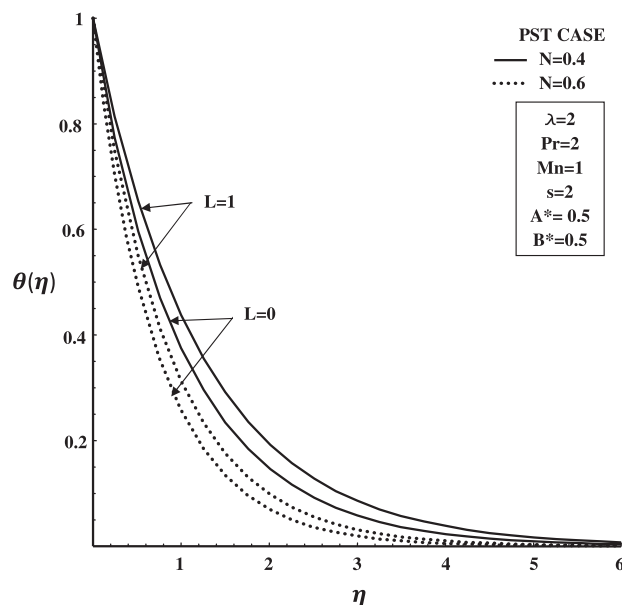


Figure 9a Effect of slip parameter L with Radiation parameter N on the temperature profile in PST case.

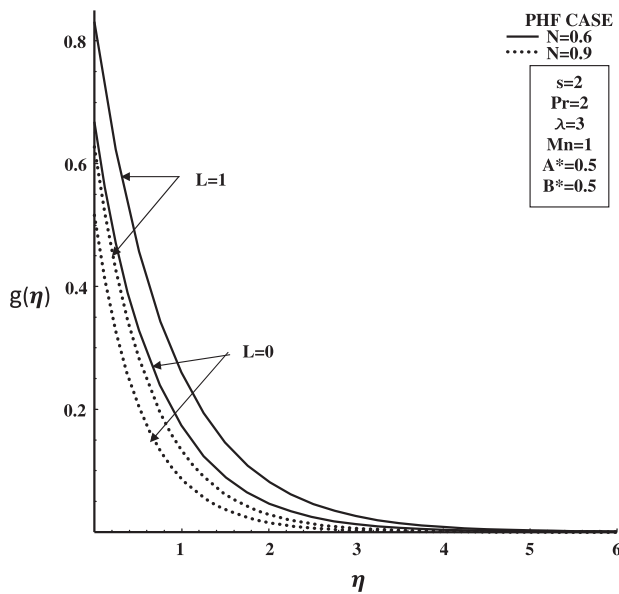


Figure 9b Effect of slip parameter L with radiation parameter N on the temperature profile in PHF case.

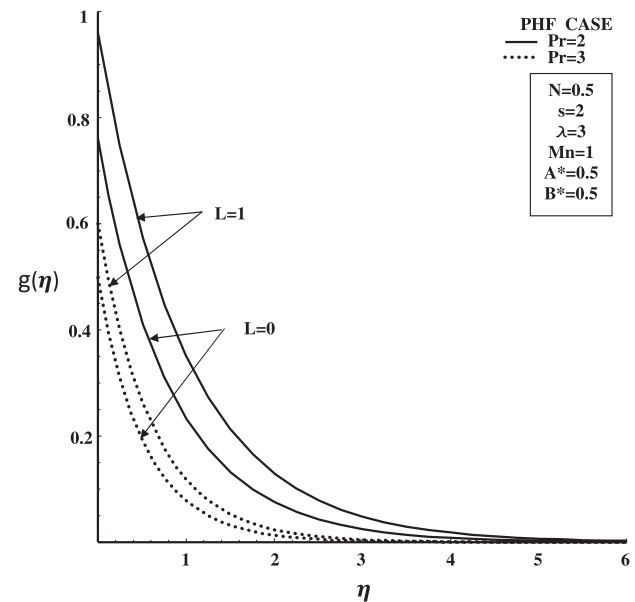


Figure 10b Effect of slip parameter L with Radiation parameter N on the temperature profile in PHF case.

illustrated in Figs. 7 and 8. The temperature rises in the case of space ($A^* > 0$ Fig. 7a) and temperature dependent ($B^* > 0$ Fig. 8a) heat source and gets reduce in the case of space ($A^* < 0$, Fig. 7b) and temperature dependent $B^* < 0$, (Fig. 8b) heat sink in both PST and PHF cases. The combined effect of slip parameter with space and temperature dependent heat source/sink parameters always lead to thickening of the thermal boundary layer.

Figs. 9 and 10 demonstrate the effects of radiation parameter and Prandtl number with ' L ' on temperature profile in PST and PHF cases respectively. The temperature profile decreases with the increasing values of radiation parameter

in both PST and PHF case (Figs. 9) and same trend is observed on the Prandtl number (Figs. 10). This is due to the fact that the thermal boundary layer thickness decreases as radiation and Prandtl number increases. The combined effect of both the parameters with partial slip parameter reduces the temperature in both PST and PHF cases.

The variation in surface velocity gradient $f_{\eta\eta}(0)$ with L, Mn, s and λ is presented in Table 2. The non-dimensional surface velocity gradient $f_{\eta\eta}(0)$ increases with L and decreases with the increasing values of Mn, s and λ . The values of wall temperature gradient $\theta_{\eta}(0)$ and wall temperature $g(0)$ for various values of parameters governing the flow are calculated and tabulated in Table 3. This table reveals that the wall temperature gradient $\theta_{\eta}(0)$ and the wall temperature $g(0)$ get increase with the increasing values of L, Mn, λ, A^* and B^* and get decrease with s, Pr and N .

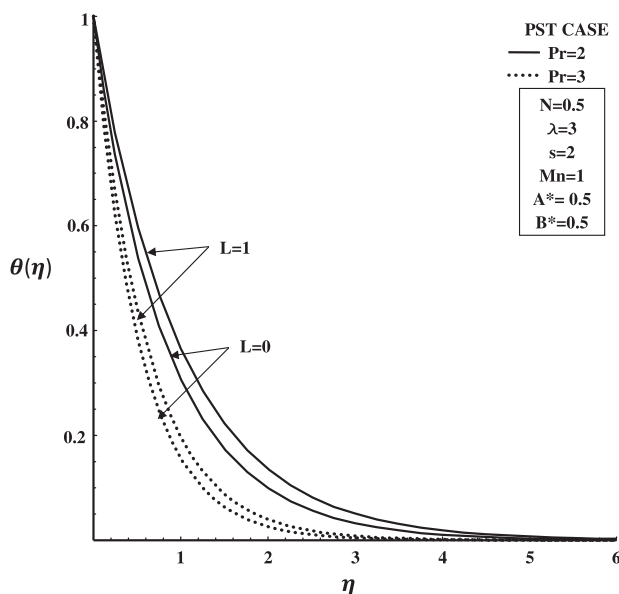


Figure 10a Effect of slip parameter L with Prandtl number Pr on the temperature profile in PST case.

Table 2 Values of $f_{\eta\eta}(0)$.

Parameter	Value	$f_{\eta\eta}(0)$
L	1	-0.766771
	2	-0.433609
	3	-0.302384
Mn	0	-0.753734
	1	-0.766771
	2	-0.777505
s	-1	-0.622797
	0	-0.675282
	1	-0.724318
λ	1	-0.737278
	3	-0.766771
	5	-0.786588

Note: While studying the effect of individual parameter the following value are assumed $L = 1, \lambda = 3, s = 2$ and $Mn = 1$.

Table 3 Values of $\theta_\eta(0)$ and $g(0)$ for various values of the governing parameter in PST and PHF cases respectively.

Parameter	Value	PST $\theta_\eta(0)$	PHF $g(0)$
L	1	-1.09552	0.91327
	2	-1.05927	0.94422
	3	-1.04460	0.95740
Mn	0	-1.10502	0.90552
	2	-1.08803	0.91948
	4	-1.07683	0.92894
λ	1	-1.11762	0.89548
	3	-1.09552	0.91327
	5	-1.08192	0.92461
s	-2	-0.09697	9.08958
	2	-1.09552	0.91327
	4	-2.18093	0.45923
Pr	2	-1.09552	0.91327
	3	-1.69666	0.59079
	4	-2.27724	0.44054
A^*	-1	-1.12094	0.89019
	0	-1.10139	0.90794
	1	-1.08184	0.92569
B^*	-0.5	-1.27812	0.78333
	0	-1.17105	0.85464
	0.5	-1.03772	0.96386
N	0.2	-0.38820	2.06838
	0.4	-0.85612	1.08993
	0.6	-1.19855	0.79819

Note: While studying the effect of individual parameter the following value are assumed $L = 1, \lambda = 3, s = 2, Mn = 1, Pr = 2, A^* = B^* = 0.3$ and $N = 0.5$.

5. Conclusion

An analysis has been carried out to study the effect of partial slip on hydromagnetic boundary layer flow over a porous stretching sheet with space and temperature dependent internal heat generation/absorption, thermal radiation and wall mass transfer (suction/blowing). The dimensionless governing equations for this investigation are solved analytically using hypergeometric functions. The conclusions derived from the present study are given below

- The velocity profile decreases with the increasing values of magnetic parameter, porosity parameter, suction parameter and increases with blowing parameter.
- The presence of partial slip parameter leads to decrease the velocity profile. The combined effect of partial slip parameter with other parameters always decreases the thickness of the momentum boundary layer.
- The increasing values of porosity, magnetic parameter and the partial slip parameters increase the temperature of the fluid in both PST and PHF cases and in turn the fluid temperature decreases when ever radiation and the Prandtl number get increase.
- The temperature of the fluid rises in the case of space and temperature dependent heat source and decreases in the case of heat sink.

- The surface velocity gradient decreases with the magnetic parameter and porosity parameter and increases with partial slip parameter. The values of wall temperature gradient and wall temperature get increase with the increasing of the slip parameter, magnetic parameter, porosity parameter, space and temperature dependent heat source parameter and decrease with Prandtl number, radiation parameter, space and temperature dependent heat sink parameter.
- The partial slip parameter always leads to thickening of the thermal boundary layer.

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